(My) Methods for Cosmic Web Analysis

Bayesian inference, Decision Theory, Information Theory

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The BORG inference framework

Bayesian Origin Reconstruction from Galaxies

- A Bayesian Hierarchical Model:

$$\mathcal{P}(\hat{\delta}) \propto \exp\left(-\frac{1}{2}\sum_k \frac{|\hat{\delta}_k|^2}{P_k}\right)$$ initial conditions

$\rho_\mathrm{m} = \mathcal{F}(\delta)$ total evolved matter density

$\rho_\mathrm{g} \propto \rho_\mathrm{m}^\alpha$ biased galaxy distribution

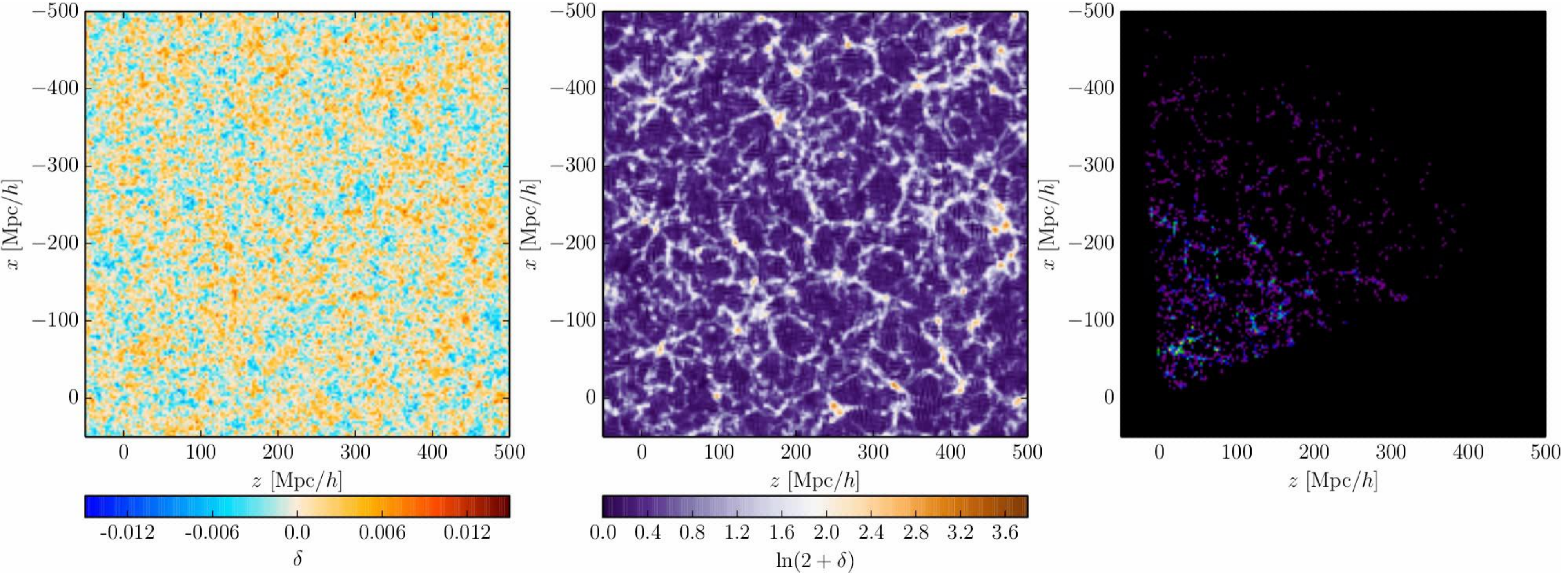
$\rho_\mathrm{g}^\mathrm{s}(\vec{x}) = S(\vec{x})\rho_\mathrm{g}(\vec{x})$ selected sample

$N_\mathrm{g} \curvearrowleft \mathcal{P}(N_\mathrm{g}|\rho_\mathrm{g}^\mathrm{s})$ galaxy number count: random extraction (Poisson)

- The multi-million dimensional posterior distribution is sampled via Hamiltonian Monte Carlo.

Jasche & Wandelt 2013, 1203.3639 – Jasche, FL & Wandelt 2015, 1409.6308 – Lavaux & Jasche 2016, 1509.05040 – Jasche & Lavaux 2018, 1806.11117

The BORG SDSS analysis

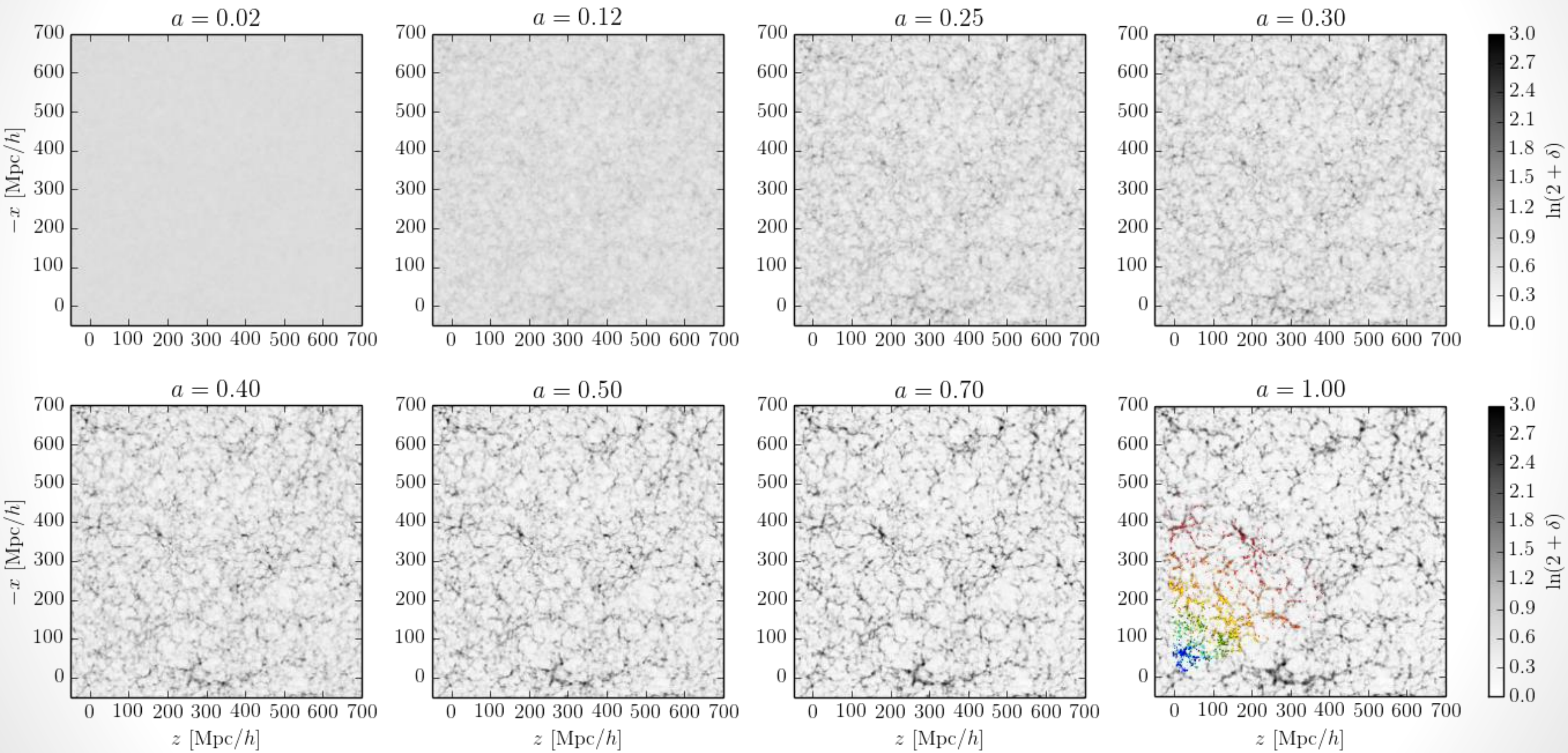


Initial conditions | Final conditions | Observations

334,074 galaxies, ≈ 17 million parameters, 3 TB of primary data products,
12,000 samples, ≈ 250,000 data model evaluations, 10 months on 32 cores

Jasche, FL & Wandelt 2015, arXiv:1409.6308

BORG infers the evolution of cosmic structure



Jasche, FL & Wandelt 2015, arXiv:1409.6308

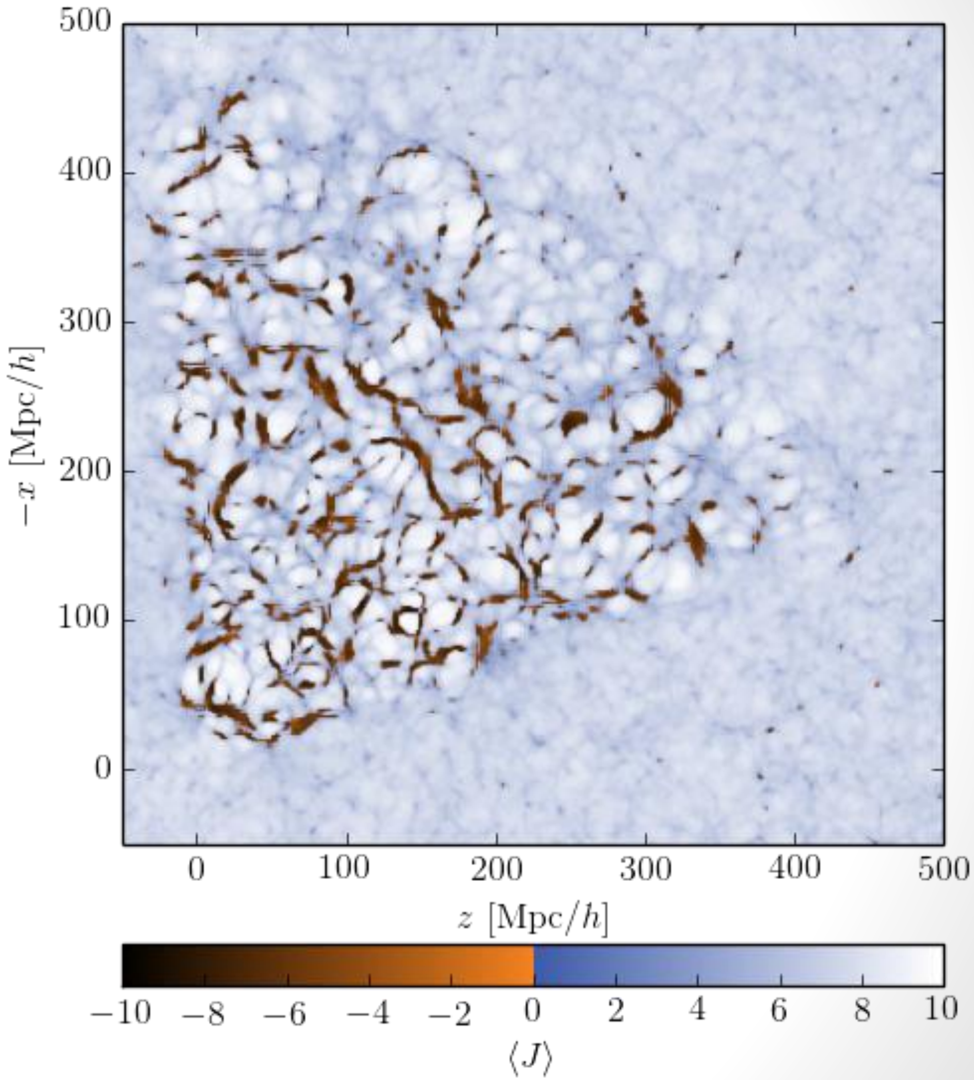
BORG infers Lagrangian dynamics in real data

$$\vec{x} = \vec{q} + \vec{\Psi}(\vec{x})$$

- The dark matter phase-space sheet has been studied so far mostly in simulations

  e.g. Abel, Hahn & Kaehler 2012, arXiv:1111.3944
  Shandarin, Habib & Heitmann 2012, arXiv:1111.2366
  Neyrinck 2012, arXiv:1202.3364
  Sousbie & Colombi 2016, arXiv:1509.07720

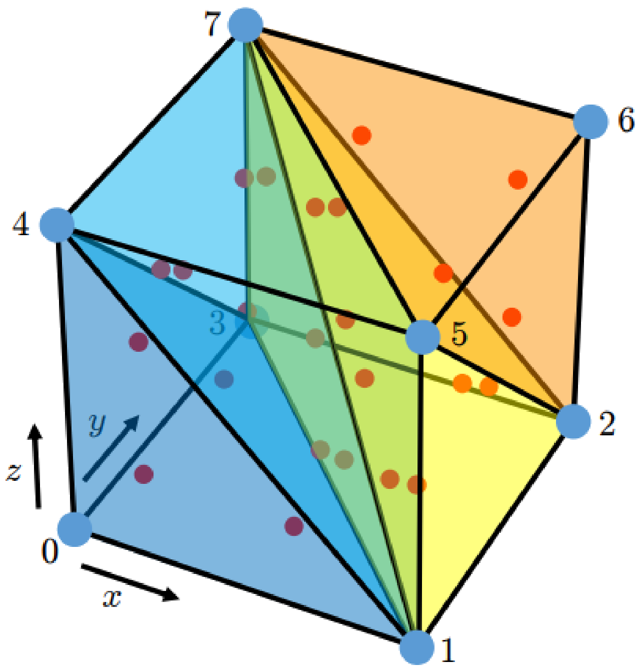
- BORG infers Lagrangian dynamics in real data
- Identified structures have a direct physical interpretation

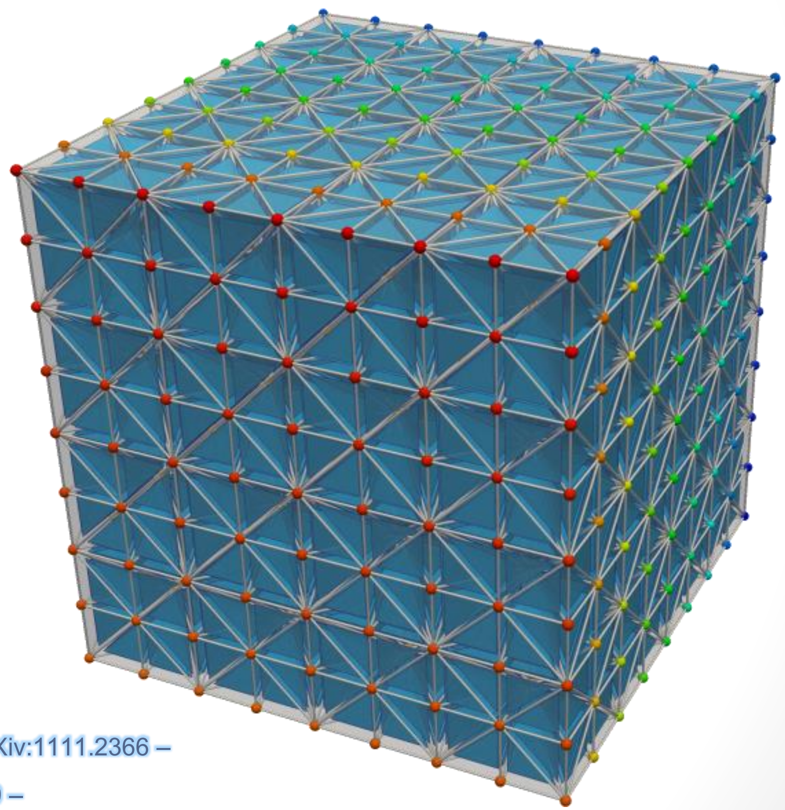


FL, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093

Tracing the dark matter phase-space sheet

- Delaunay tessellation of elementary Lagrangian cubes

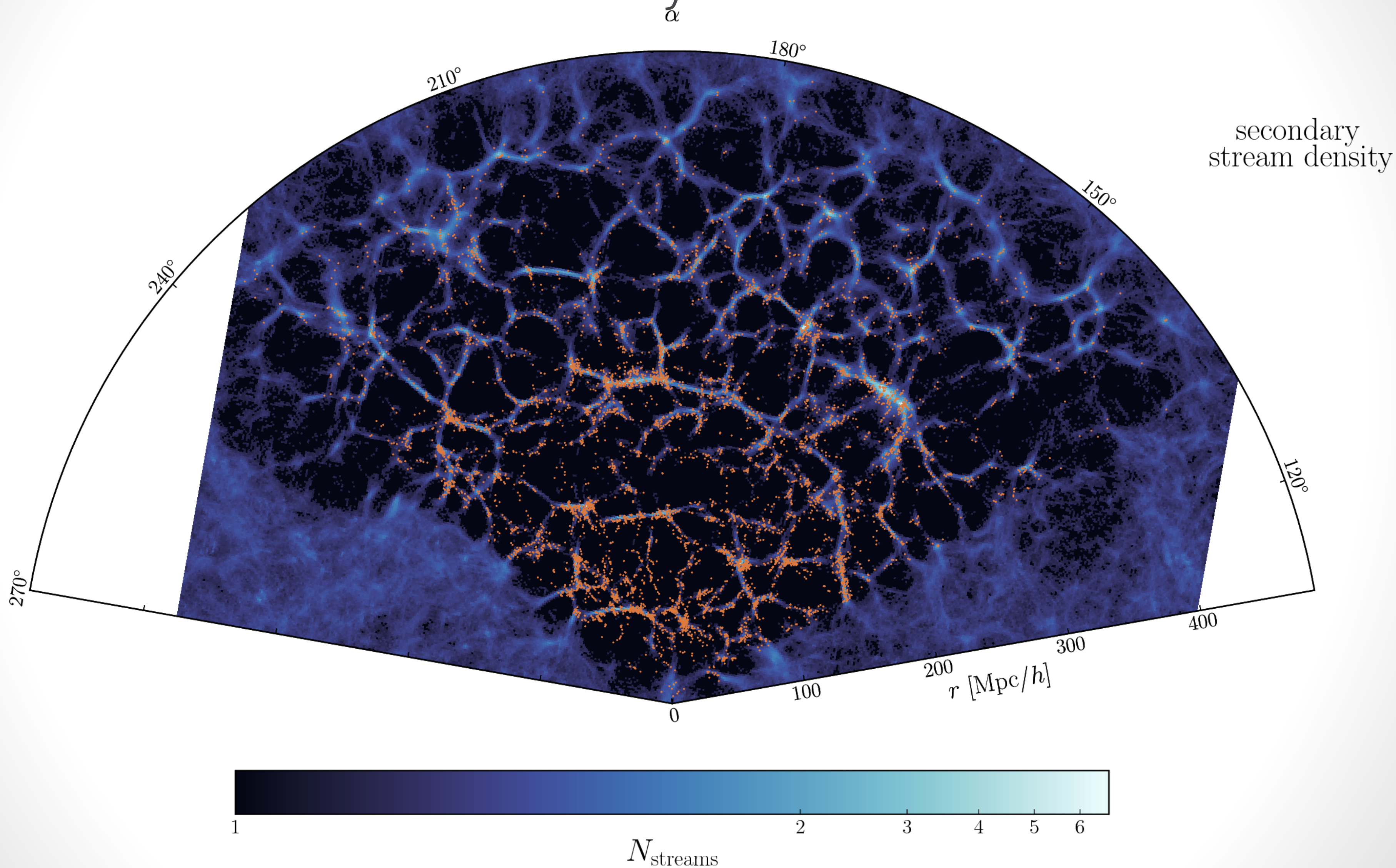




Abel, Hahn & Kaehler 2012, arXiv:1111.3944 – Shandarin, Habib & Heitmann 2012, arXiv:1111.2366 –
Hahn, Abel & Kaehler 2013, arXiv:1210.6652 – Hahn & Angulo 2016, arXiv:1501.01959 –
Sousbie & Colombi 2016, arXiv:1509.07720

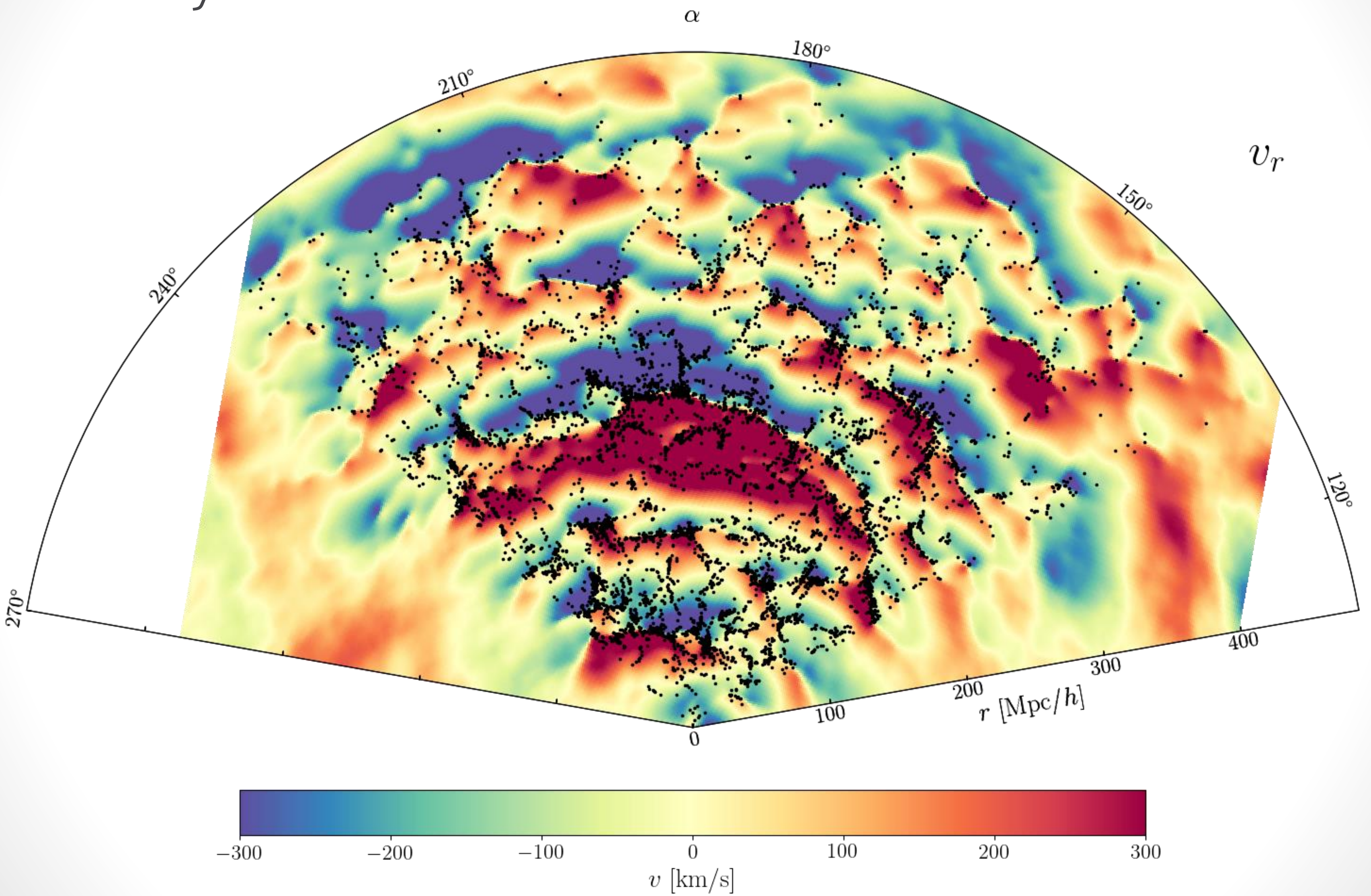
FL, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093

Dark matter stream density



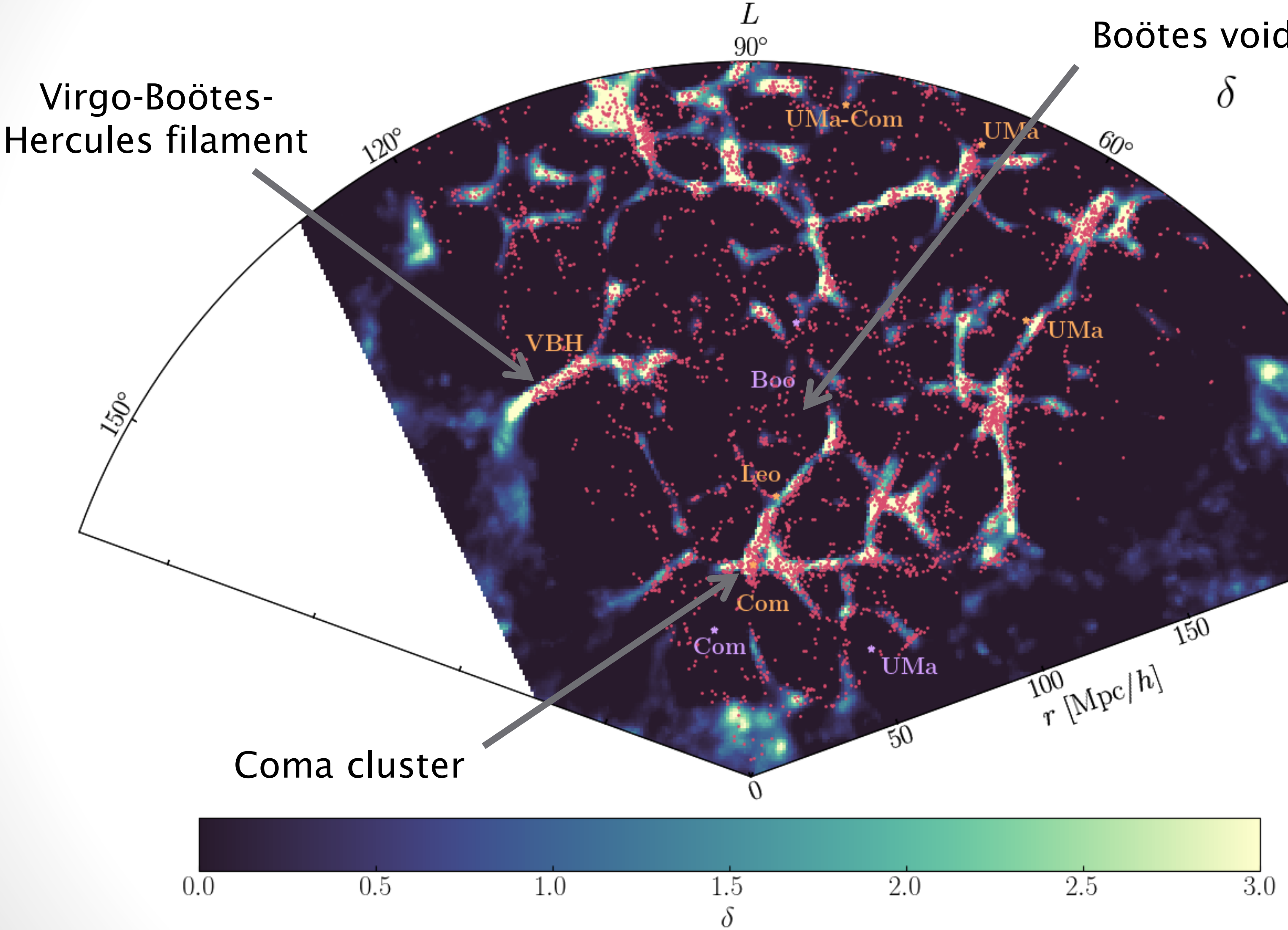
FL, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093

Velocity field



FL, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093

Cosmography in the supergalactic plane



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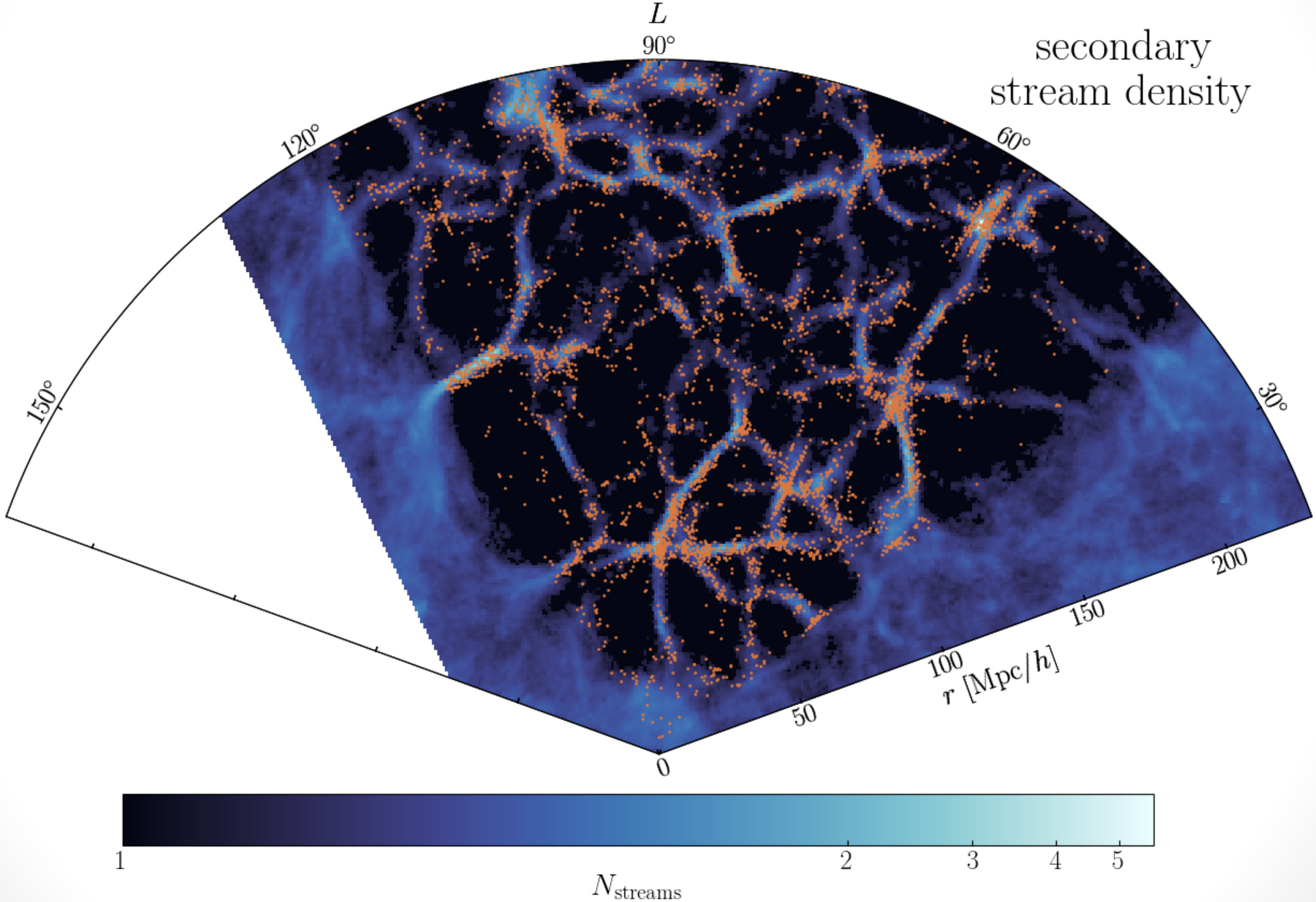

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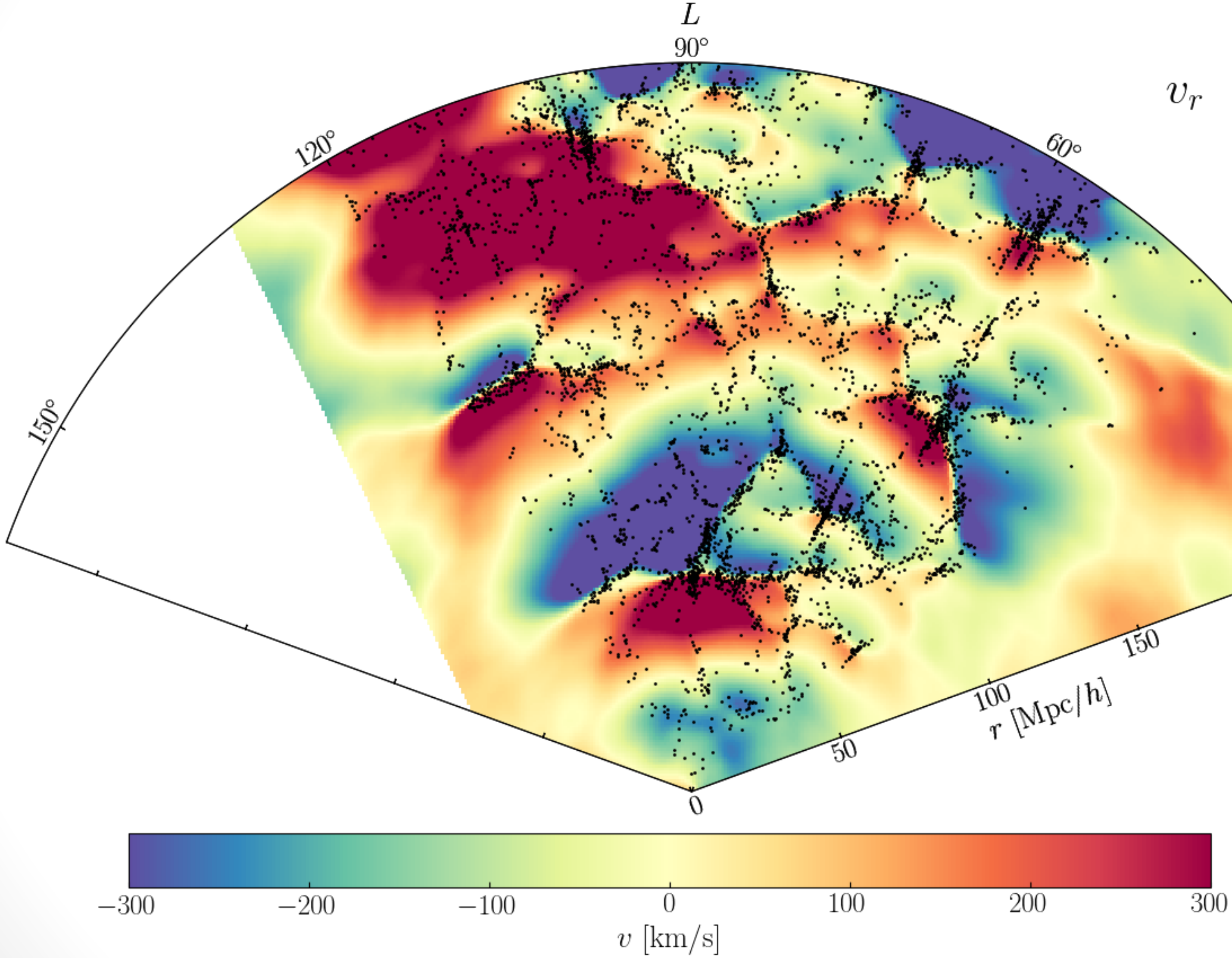
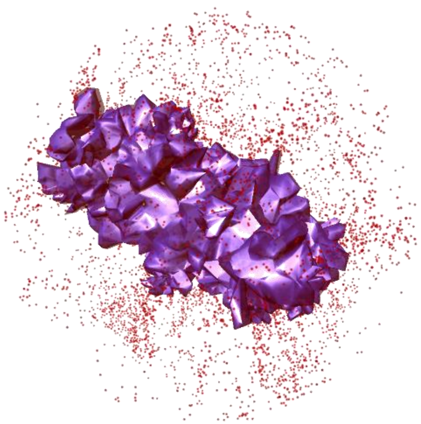


FL, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093

Cosmic web elements: some algorithms

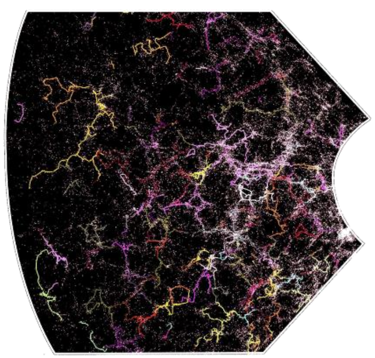
- "**Structure finders**" focus on one element at a time
  - ZOBOV/VIDE

Neyrinck 2008, arXiv:0712.3049
Sutter *et al.* 2015, arXiv:1406.1191



  - DisPerSE

Sousbie 2011, arXiv:1009.4015
Sousbie *et al.* 2011, arXiv:1009.4014



- "**Classifiers**" dissect the cosmic web all at once
  - The T-web (tidal field tensor)

Hahn et al. 2007, arXiv:astro-ph/0610280

  - DIVA (Lagrangian displacement field, potential structures)

Lavaux & Wandelt 2010, arXiv:0906.4101

  - ORIGAMI (particle crossings)

Falck, Neyrinck & Szalay 2012, arXiv:1201.2353

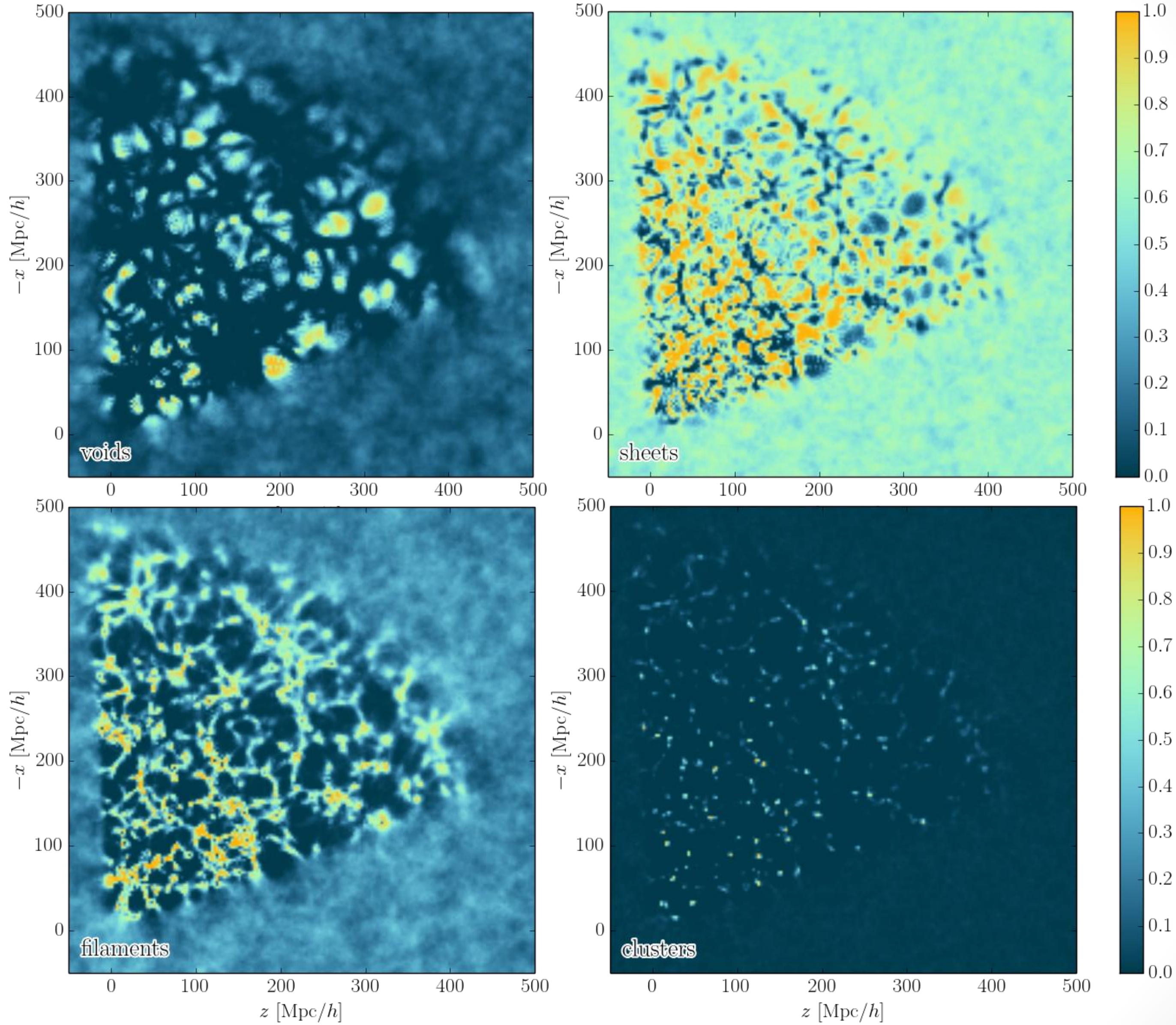
  - LICH (Lagrangian displacement field, potential and vortical structures)

FL, Jasche, Lavaux, Wandelt & Percival 2017

and many others…

T-web structures inferred by BORG

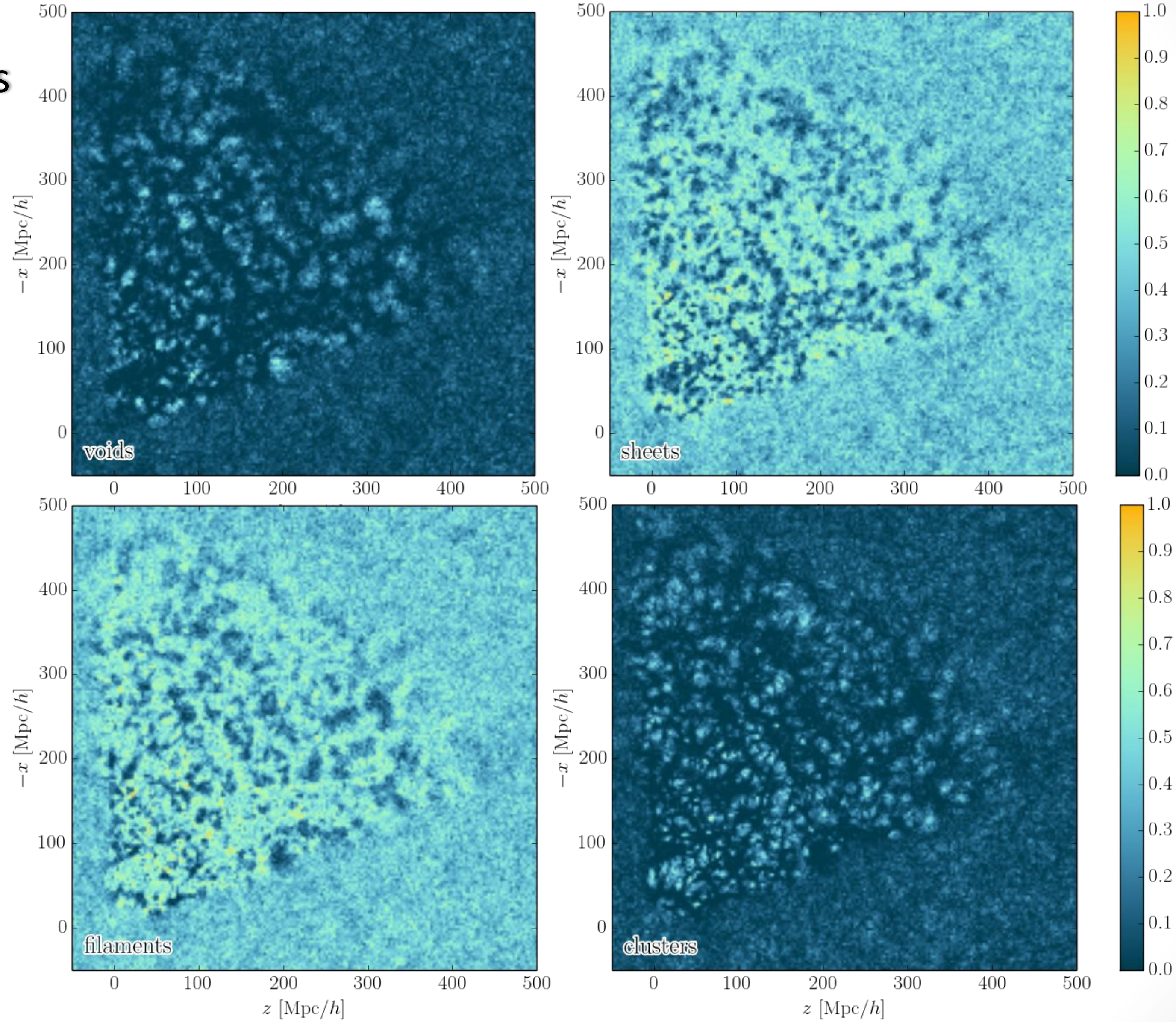
Final conditions



FL, Jasche & Wandelt 2015, arXiv:1502.02690

T-web structures inferred by BORG

Initial conditions



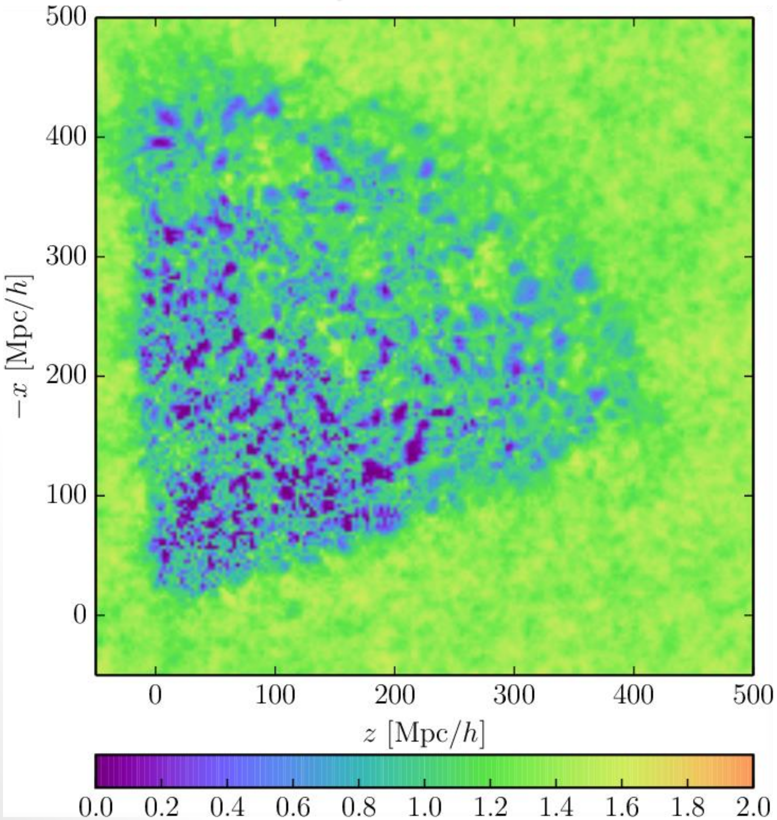
FL, Jasche & Wandelt 2015, arXiv:1502.02690

What is the information content of these maps?

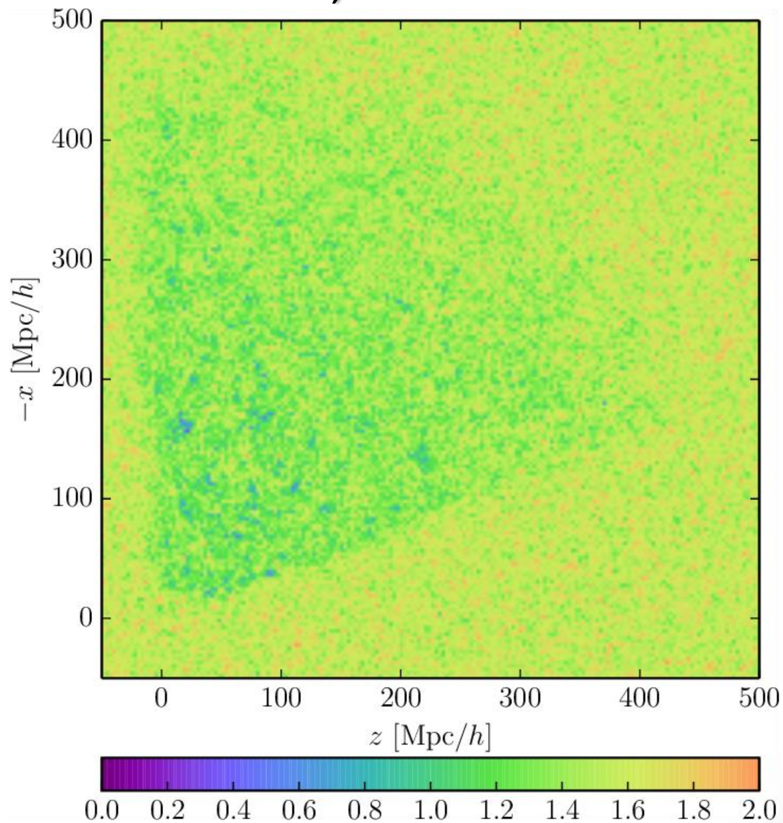
- Shannon entropy

$$H\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d)\right] \equiv -\sum_{i=0}^{3} \mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)\log_2(\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d))$$ in shannons (Sh)

T-web, final conditions



T-web, initial conditions



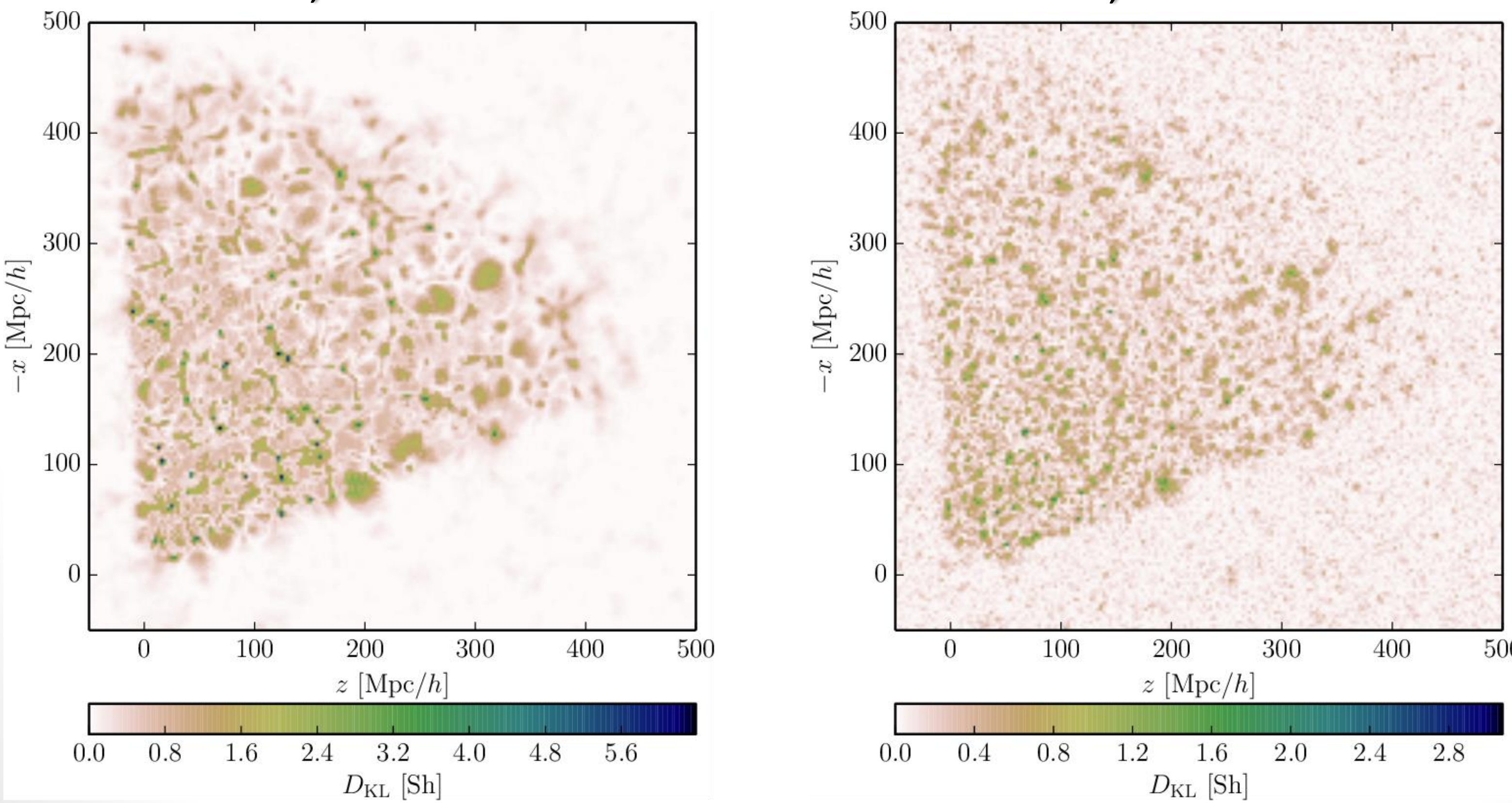
FL, Jasche & Wandelt 2015, arXiv:1502.02690

How much did the data surprise us?

- information gain a.k.a. relative entropy or Kullback-Leibler divergence

$$D_{\mathrm{KL}}\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d)||\mathcal{P}(\mathrm{T})\right] = \sum_i \mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)\log_2\left(\frac{\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)}{\mathcal{P}(\mathrm{T}_i)}\right)$$ in Sh

T-web, final conditions **T-web, initial conditions**



FL, Jasche & Wandelt 2015, arXiv:1502.02690

A decision rule for structure classification

- Space of "input features":

$$\{\mathrm{T}_0 = \text{void}, \mathrm{T}_1 = \text{sheet}, \mathrm{T}_2 = \text{filament}, \mathrm{T}_3 = \text{cluster}\}$$

- Space of "actions":

$$\{a_0 = \text{``decide void''}, a_1 = \text{``decide sheet''}, a_2 = \text{``decide filament''}, a_3 = \text{``decide cluster''}, a_{-1} = \text{``do not decide''}\}$$

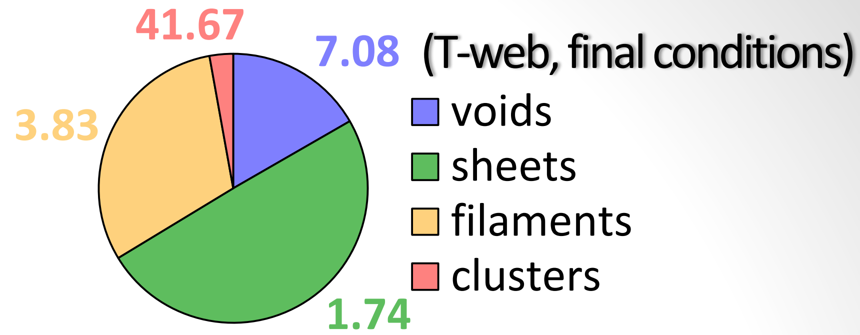
➡ A problem of Bayesian decision theory:
one should take the action that maximizes the utility

$$U(a_j(\vec{x}_k)|d) = \sum_{i=0}^{3} G(a_j|\mathrm{T}_i)\,\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)$$

- How to write down the gain functions?

FL, Jasche & Wandelt 2015, arXiv:1503.00730

Gambling with the Universe



- One proposal:

$$G(a_j|\mathrm{T}_i) = \begin{cases} \dfrac{1}{\mathcal{P}(\mathrm{T}_i)} - \alpha & \text{if } j \in [\![0,3]\!] \text{ and } i = j \\ -\alpha & \text{if } j \in [\![0,3]\!] \text{ and } i \neq j \\ 0 & \text{if } j = -1. \end{cases}$$

"Winning"

"Losing"

"Not playing"

- Without data, the expected utility is

$$U(a_j) = 1 - \alpha \quad \text{if } j \neq 1$$

"Playing the game"

$$U(a_{-1}) = 0$$

"Not playing the game"

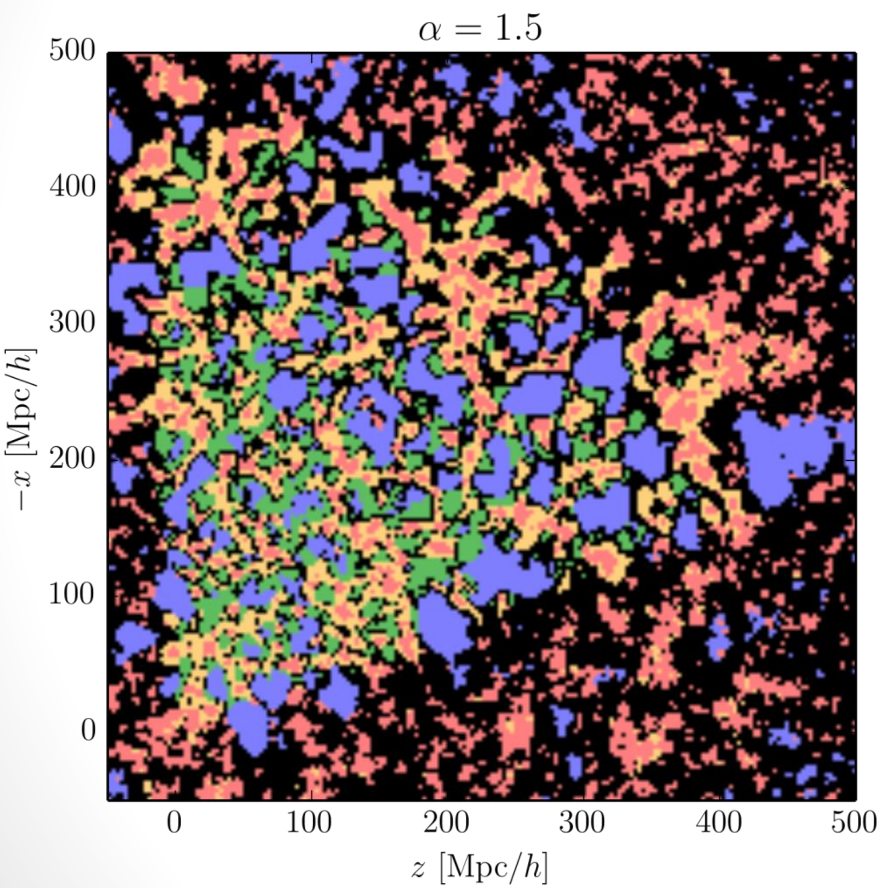
- With $\alpha = 1$, it's a *fair game* always play
  "speculative map" of the LSS
- Values $\alpha > 1$ represent an *aversion for risk*
  increasingly "conservative maps" of the LSS

FL, Jasche & Wandelt 2015, arXiv:1503.00730

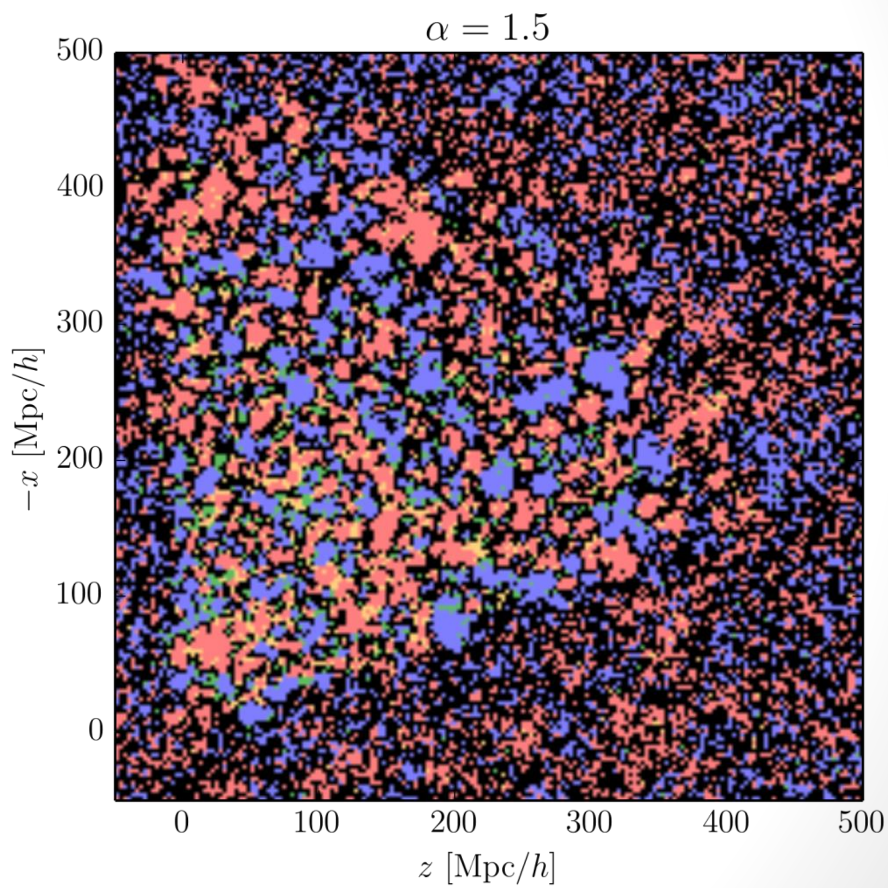
Playing the game…

voids | filaments | undecided
sheets | clusters

Final conditions

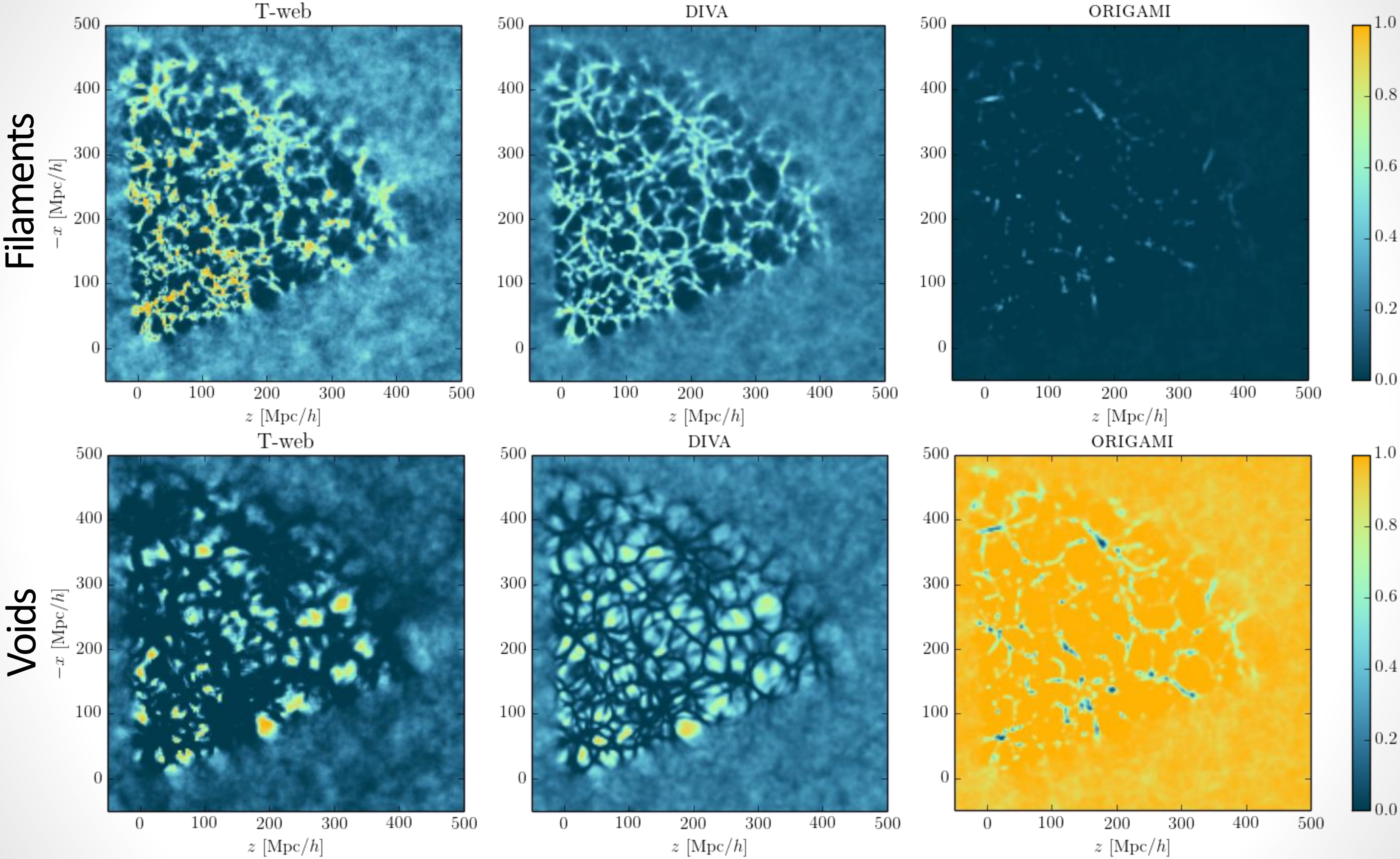


Initial conditions



FL, Jasche & Wandelt 2015, arXiv:1503.00730

Comparing classifiers



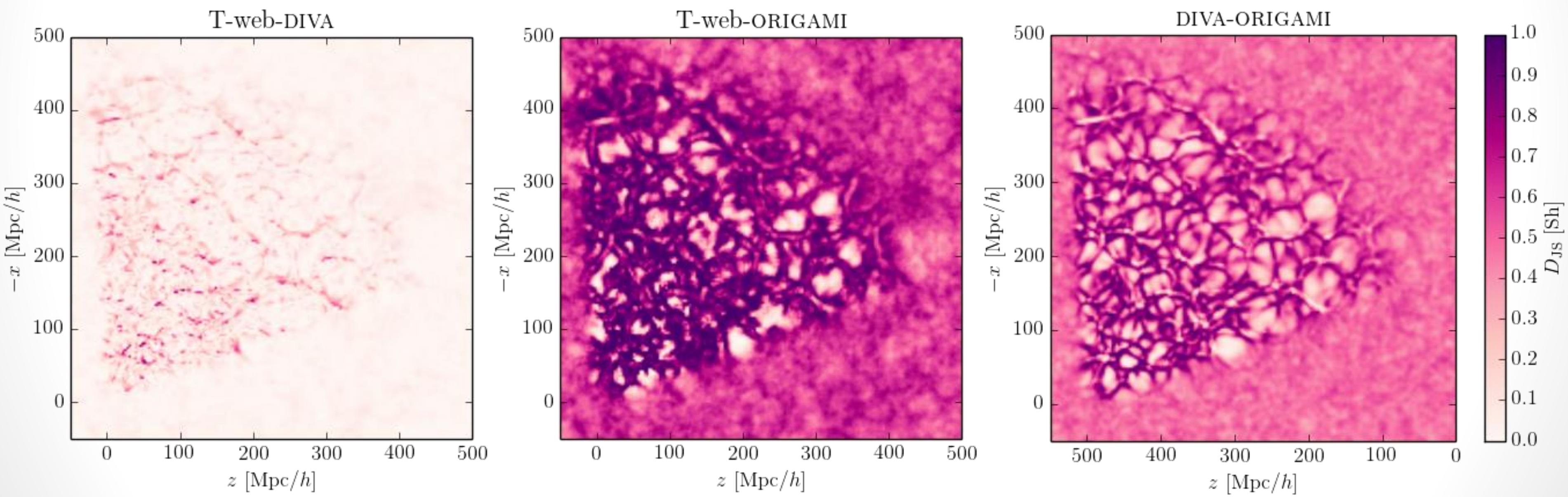
FL, Jasche & Wandelt 2015, arXiv:1502.02690

FL, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093

How similar are different classifications?

- Jensen-Shannon divergence

$$D_{\mathrm{JS}}[\mathcal{P}:\mathcal{Q}] \equiv \frac{1}{2} D_{\mathrm{KL}}\left[\mathcal{P} || \frac{\mathcal{P}+\mathcal{Q}}{2}\right] + \frac{1}{2} D_{\mathrm{KL}}\left[\mathcal{Q} || \frac{\mathcal{P}+\mathcal{Q}}{2}\right]$$



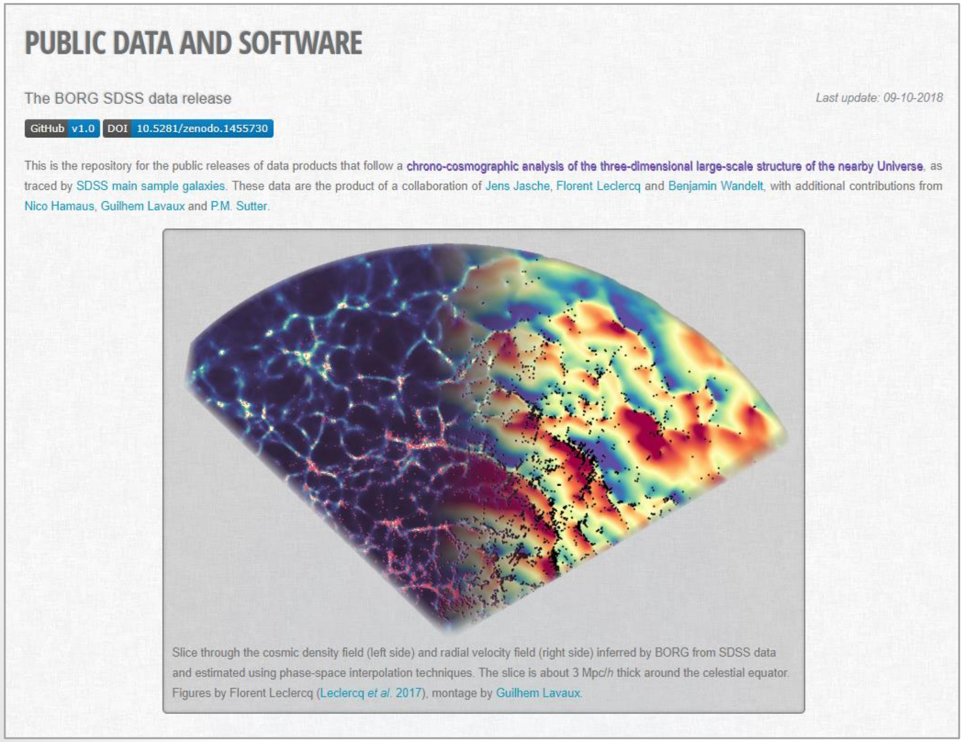
(more about the Jensen-Shannon divergence later)

FL, Lavaux, Jasche & Wandelt 2016, 1606.06758

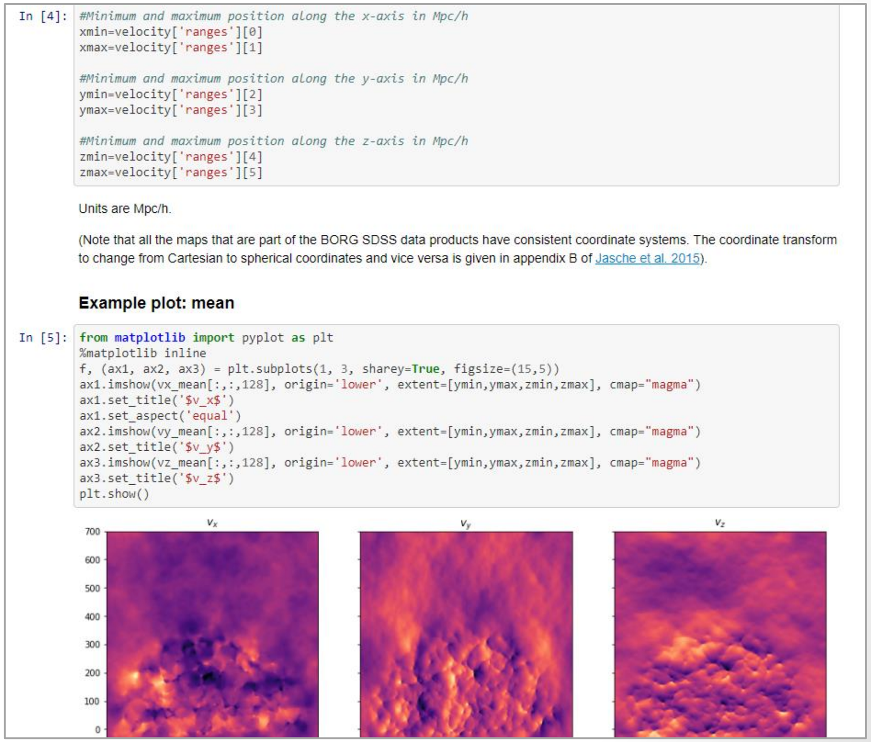
The BORG SDSS data release

- All data products are publicly available: doi: 10.5281/zenodo.1455729

http://data.florent-leclercq.eu/



https://github.com/florent-leclercq/borg_sdss_data_release



Conclusions

- BORG is a Bayesian inference engine allowing the analysis of the large-scale structure and its formation history.
- Thanks to BORG, the cosmic web can be described using various classifiers.
- A probabilistic analysis of the cosmic web yields a data-supported connection between cosmology and information theory.
- Decision theory offers a framework to choose between different classifiers, with utility functions depending on the desired use.